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Vacuum-induced stationary entanglement in radiatively coupled three-level atoms

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Abstract
We consider a pair of three-level atoms interacting with a common vacuum and analyse the process of entanglement production due to spontaneous emission. We show that in the case of closely separated atoms collective damping can generate robust entanglement of the asymptotic states.

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1. Introduction

The important problem of the evolution of entanglement in realistic quantum systems interacting with their environments was mainly discussed in the case of two two-level systems (qubits). In that case, the interesting idea that dissipation can create rather than destroy the entanglement was studied in detail. In particular, in the case of two-level atoms, the possible production of robust or transient entanglement induced by the process of spontaneous emission was shown [1–4].

Much more complex and interesting is the process of creation of entanglement involving multilevel atoms. In such a case, quantum interference between different radiative transitions can influence the dynamics of the system. For a pair of largely separated three-level atoms, the role of such interference in the process of degradation of entanglement was studied in [5]. When the interatomic distance is comparable to the wavelength of the emitted radiation, the coupling between the atoms via a common vacuum gives rise to the collective effects such as collective damping and dipole–dipole interaction. Such effects are well known [6], particularly in the case of two-level atoms. In the system of three-level atoms having closely lying excited states, radiative coupling can produce a new interference effect in the spontaneous emission. This effect manifests itself by the cross-coupling between radiation transitions with orthogonal dipole moments [7]. All such collective properties of the system influence the entanglement between three-level atoms.

In the paper, we study the entanglement production between three-level atoms due to collective damping (a detailed analysis of the entanglement evolution in the presence of all collective effects will be presented elsewhere). In that case, the analysis is involved since there is no simple necessary and sufficient condition of entanglement for a pair of d-level systems with $d \geqslant 3$. The Peres–Horodecki separability criterion [8, 9] only shows that states which are not positive after partial transposition (NPPT states) are entangled. But there can exist entangled states which are positive after this operation [10] (bound entangled PPT states). The problem of existence of bound entangled (i.e. non-distillable [11]) states can be analysed in terms of the rank of the density matrix of the bipartite system and the ranks of its partial traces. If a state is separable or bound entangled, then its rank must be larger than the ranks of partial traces [12]. In the paper we focus on the possibility of creation of NPPT 'free' entangled states, so we do not discuss these problems. To detect and quantify entanglement, we use the negativity of partial transposition of the density matrix.

As we show, in the limit of small separation between the atoms, the process of photon exchange between the atoms produces such correlations that the dynamics is not ergodic and there are nontrivial asymptotic stationary states. We compute the explicit form of the asymptotic state for any initial state and show that some of the asymptotic states are NPPT states, even if the initial states were PPT states. This effect occurs, for example, for a large class of diagonal, i.e. separable, initial states. We also give the example of a bound entangled PPT state which evolves into the NPPT (i.e. distillable) asymptotic state.

2. Dynamical evolution of two three-level atoms

Consider two identical three-level atoms (A and B) in the V configuration. The atoms have two near-degenerate excited states $|1_\mu\rangle$, $|2_\mu\rangle$ ($\mu = \mathrm{A}, \mathrm{B}$) and ground states $|3_\mu\rangle$. Assume that the atoms interact with the common vacuum and that the transition dipole moments of atom A are parallel to the transition dipole moments of atom B. Due to this interaction, the process of spontaneous emission from two excited levels to the ground state takes place in each individual atom, but a direct transition between excited levels is not possible. Moreover, the coupling between two atoms can be produced by the exchange of the photons. The evolution of an atomic system can be described by the following master equation [6]:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = (L^{\mathrm{A}} + L^{\mathrm{B}} + L^{\mathrm{AB}})\rho, \tag{2.1}$$

where for $\mu = \mathrm{A}, \mathrm{B}$ we have

$$L^\mu\rho = \gamma_{13}\big(2\sigma_{31}^\mu\rho\sigma_{13}^\mu - \sigma_{13}^\mu\sigma_{31}^\mu\rho - \rho\sigma_{13}^\mu\sigma_{31}^\mu\big) + \gamma_{23}\big(2\sigma_{32}^\mu\rho\sigma_{23}^\mu - \sigma_{23}^\mu\sigma_{32}^\mu\rho - \rho\sigma_{23}^\mu\sigma_{32}^\mu\big) \tag{2.2}$$

and

$$\begin{aligned} L^{\mathrm{AB}}\rho = \Gamma_{13}\big(&2\sigma_{31}^{\mathrm{A}}\rho\sigma_{13}^{\mathrm{B}} - \sigma_{13}^{\mathrm{B}}\sigma_{31}^{\mathrm{A}}\rho - \rho\sigma_{13}^{\mathrm{B}}\sigma_{31}^{\mathrm{A}} + 2\sigma_{31}^{\mathrm{B}}\rho\sigma_{13}^{\mathrm{A}} - \sigma_{13}^{\mathrm{A}}\sigma_{31}^{\mathrm{B}}\rho - \rho\sigma_{13}^{\mathrm{A}}\sigma_{31}^{\mathrm{B}}\big) \\ &+ \mathrm{i}\Omega_{13}\big[\sigma_{13}^{\mathrm{A}}\sigma_{31}^{\mathrm{B}} + \sigma_{13}^{\mathrm{B}}\sigma_{31}^{\mathrm{A}}, \rho\big] + \Gamma_{23}\big(2\sigma_{32}^{\mathrm{A}}\rho\sigma_{23}^{\mathrm{B}} - \sigma_{23}^{\mathrm{B}}\sigma_{32}^{\mathrm{A}}\rho - \rho\sigma_{23}^{\mathrm{B}}\sigma_{32}^{\mathrm{A}} \\ &+ 2\sigma_{32}^{\mathrm{B}}\rho\sigma_{23}^{\mathrm{A}} - \sigma_{23}^{\mathrm{A}}\sigma_{32}^{\mathrm{B}}\rho - \rho\sigma_{23}^{\mathrm{A}}\sigma_{32}^{\mathrm{B}}\big) + \mathrm{i}\Omega_{23}\big[\sigma_{23}^{\mathrm{A}}\sigma_{32}^{\mathrm{B}} + \sigma_{23}^{\mathrm{B}}\sigma_{32}^{\mathrm{A}}, \rho\big]. \end{aligned} \tag{2.3}$$

In equations (2.2) and (2.3), σ_{jk}^μ is the transition operator from $|k_\mu\rangle$ to $|j_\mu\rangle$ ($\mu = \mathrm{A}, \mathrm{B}$) and the coefficient γ_{j3} represents the single-atom spontaneous-decay rate from the state $|j\rangle$ ($j = 1, 2$) to the state $|3\rangle$. The coefficients Γ_{j3} and Ω_{j3} are related to the coupling between two atoms and are the collective damping and the dipole–dipole interaction potential, respectively. As was shown in [7], in such an atomic system there is also possible the radiative process in which atom A in the excited state $|1_{\mathrm{A}}\rangle$ loses its excitation which in turn excites atom B to the state $|2_{\mathrm{B}}\rangle$. This cross-coupling between two atoms is sensitive to the orientation of the transition dipole moments of atoms, and in the present paper we study the model in which that coupling is absent. We also assume that the spontaneous-decay rates satisfy

$$\gamma_{12} \approx \gamma_{13} = \gamma. \tag{2.4}$$

The remaining coefficients in equation (2.3) can be written as

$$\Gamma_{j3} = \gamma G_{j3}(R), \qquad \Omega_{j3} = \gamma F_{j3}(R), \tag{2.5}$$

where $j = 1, 2$ and R is the distance between the atoms. A detailed form of the functions $G_{j3}(R)$ and $F_{j3}(R)$ depends on the geometry of the system [7], but in general, for $R \to \infty$

$$G_{j3}(R), F_{j3}(R) \to 0$$

and for $R \to 0$

$$G_{j3}(R) \to 1,$$

whereas the function $F_{j3}(R)$ diverges.

The time evolution of the initial state ρ of the atomic system is given by the semi-group $\{T_t\}_{t\geqslant 0}$ of completely positive linear mappings acting on density matrices [13], generated by $L^{\mathrm{A}} + L^{\mathrm{B}} + L^{\mathrm{AB}}$. The properties of this semi-group crucially depend on the distance R between the two atoms. As can be shown by a direct calculation, when the distance is large (compared to the radiation wavelength), the semi-group $\{T_t\}_{t\geqslant 0}$ is uniquely relaxing with the asymptotic state $|3_{\mathrm{A}}\rangle \otimes |3_{\mathrm{B}}\rangle$. On the other hand, when R is small, $\Gamma_{13}, \Gamma_{23} \to \gamma$ and Ω_{13}, Ω_{23} are large, so we can use the approximation

$$\Gamma_{13} = \Gamma_{23} = \gamma \quad \text{and} \quad \Omega_{13} = \Omega_{23} = \Omega. \tag{2.6}$$

In that case, the semi-group is not uniquely relaxing and asymptotic stationary states are nontrivial and depend on initial conditions.

We do not discuss the details of the time evolution of the system, but we focus on the analysis of the asymptotic behaviour of the dynamics of atoms with a small separation, when conditions (2.4) and (2.6) are satisfied. The master equation (2.1) can be used to obtain differential equations for the matrix elements of any state ρ. We consider the matrix elements of ρ with respect to the basis of $\mathbb{C}^3 \otimes \mathbb{C}^3$ given by vectors

$$|j_{\mathrm{A}}\rangle \otimes |k_{\mathrm{B}}\rangle, \qquad j, k = 1, 2, 3 \tag{2.7}$$

taken in the lexicographic order. The equations for $\rho_{lm}, l, m = 1, \ldots, 9$ form a system of linear differential equations which can be solved by elementary methods. Using these solutions, after a long calculation, we obtain the explicit form of the asymptotic state ρ_{as} for any initial state ρ with the matrix elements ρ_{lm}:

$$\rho_{\mathrm{as}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & z & -x & -z & w \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{z} & 0 & 0 & y & -\overline{z} & -y & v \\ 0 & 0 & -x & 0 & 0 & -z & x & z & -w \\ 0 & 0 & -\overline{z} & 0 & 0 & -y & \overline{z} & y & -v \\ 0 & 0 & \overline{w} & 0 & 0 & \overline{v} & -\overline{w} & -\overline{v} & t \end{pmatrix}, \tag{2.8}$$

where

$$\begin{aligned}
x &= \tfrac{1}{8}(\rho_{22} + 2\rho_{33} + \rho_{44} + 2\rho_{77} - 2\,\mathrm{Re}\,\rho_{24} - 4\,\mathrm{Re}\,\rho_{37}),\\
z &= \tfrac{1}{4}(\rho_{36} - \rho_{38} - \rho_{76} + \rho_{78}),\\
w &= \tfrac{1}{4}(\rho_{26} + \rho_{28} + 2\rho_{39} - \rho_{46} - \rho_{48} - 2\rho_{79}),\\
y &= \tfrac{1}{8}(\rho_{22} + \rho_{44} + 2\rho_{66} + 2\rho_{88} - 2\,\mathrm{Re}\,\rho_{24} - 4\,\mathrm{Re}\,\rho_{68}),\\
v &= \tfrac{1}{4}(-\rho_{23} - \rho_{27} + \rho_{43} + \rho_{47} + 2\rho_{69} - 2\rho_{89})
\end{aligned} \tag{2.9}$$

and

$$t = 1 - 2x - 2y.$$

To get some insight into the process of creation of the nontrivial asymptotic state ρ_{as}, it may be useful to consider the basis of collective states in $\mathbb{C}^9$, given by the doubly excited states

$$|e_1\rangle = |1_\mathrm{A}\rangle \otimes |1_\mathrm{B}\rangle, \qquad |e_2\rangle = |2_\mathrm{A}\rangle \otimes |2_\mathrm{B}\rangle,$$

the ground state

$$|g\rangle = |3_\mathrm{A}\rangle \otimes |3_\mathrm{B}\rangle$$

and generalized symmetric and antisymmetric Dicke states (see, e.g. [14])

$$\begin{aligned}
|s_{kl}\rangle &= \tfrac{1}{\sqrt{2}}[|k_\mathrm{A}\rangle \otimes |l_\mathrm{B}\rangle + |l_\mathrm{A}\rangle \otimes |k_\mathrm{B}\rangle],\\
|a_{kl}\rangle &= \tfrac{1}{\sqrt{2}}[|k_\mathrm{A}\rangle \otimes |l_\mathrm{B}\rangle - |l_\mathrm{A}\rangle \otimes |k_\mathrm{B}\rangle],
\end{aligned} \tag{2.10}$$

where $k, l = 1, 2, 3; k < l$. The states (2.10) are entangled, but in contrast to the case of two-level atoms, they are not maximally entangled. One can also check that the doubly excited states $|e_1\rangle, |e_2\rangle$ and the symmetric Dicke states $|s_{kl}\rangle$ decay to the ground state $|g\rangle$, whereas antisymmetric states $|a_{13}\rangle$ and $|a_{23}\rangle$ decouple from the environment and therefore are stable. Moreover, the state $|a_{12}\rangle$ is not stable, but is asymptotically nontrivial. Note that the collective states can be used to the direct characterization of the asymptotic behaviour of the system. In particular, the parameters x and y in (2.8) are given by the populations in the antisymmetric states $|a_{13}\rangle, |a_{23}\rangle$ and $|a_{12}\rangle$:

$$\begin{aligned}
x &= \tfrac{1}{4}(\langle a_{12}|\rho|a_{12}\rangle + 2\langle a_{13}|\rho|a_{13}\rangle),\\
y &= \tfrac{1}{4}(\langle a_{12}|\rho|a_{12}\rangle + 2\langle a_{23}|\rho|a_{23}\rangle).
\end{aligned}$$

The remaining parameters can be computed in terms of the coherences between the collective states. Since the populations $\langle a_{13}|\rho|a_{13}\rangle$ and $\langle a_{23}|\rho|a_{23}\rangle$ are stationary, the states which have the property of trapping the initial populations in $|a_{13}\rangle$ or $|a_{23}\rangle$, create the nontrivial asymptotic state ρ_{as} with the stationary entanglement. On the other hand, the population in the state $|a_{12}\rangle$ is not stable, but can be transformed into $\langle a_{13}|\rho|a_{13}\rangle$ and $\langle a_{23}|\rho|a_{23}\rangle$ in such a way that the values of the parameters x and y are fixed. The explicit examples of such a behaviour of the system will be discussed in the following section.

3. Generation of NPPT states

To describe the process of the creation of correlation between two atoms leading to their entanglement, we need the effective measure of mixed-states entanglement. For such a measure one usually takes the entanglement of the formation $E_F(\rho)$ [15], but in practice it is not known how to compute this measure for the pairs of d-level systems in the case when $d > 2$. A computable measure of entanglement proposed in [16] is based on the trace norm of the partial transposition ρ^{PT} of the state ρ. From the Peres–Horodecki criterion of separability

[8, 9], it follows that if ρ^{PT} is not positive, then ρ is not separable and one defines the *negativity* of the state ρ as

$$N(\rho) = \frac{||\rho^{\mathrm{PT}}|| - 1}{2}. \tag{3.1}$$

$N(\rho)$ is equal to the absolute value of the sum of the negative eigenvalues of ρ^{PT} and is an entanglement monotone, but it cannot detect bound entangled states [10]. Using measure (3.1), one can check that generalized Dicke states are indeed not maximally entangled, since

$$N(|s_{kl}\rangle) = N(|a_{kl}\rangle) = \tfrac{1}{2}.$$

In this section, we study the negativity of the asymptotic states (2.8). For such initial states, where only the populations in antisymmetric Dicke states are nonzero, it is possible to obtain an analytic expression for asymptotic negativity. By a direct calculation one shows that

$$N(\rho_{\mathrm{as}}) = \tfrac{1}{2}\left[\sqrt{4(x^2+y^2)+t^2} - t\right]. \tag{3.2}$$

Note that every nontrivial asymptotic state from this class is entangled. On the other hand, the asymptotic negativity for initial states with nonzero coherences can only be studied numerically.

3.1. Pure separable initial states

We start with pure states (2.7). It is obvious that the initial states $|1_{\mathrm{A}}\rangle \otimes |1_{\mathrm{B}}\rangle$ and $|2_{\mathrm{A}}\rangle \otimes |2_{\mathrm{B}}\rangle$ decay to the ground state $|g\rangle$. On the other hand, the initial state $|1_{\mathrm{A}}\rangle \otimes |3_{\mathrm{B}}\rangle$ (atom A in the excited state and atom B in the ground state) has the population in the Dicke state $|a_{13}\rangle$ which is equal to $\frac{1}{2}$, thus for that state

$$x = \tfrac{1}{4}, \qquad t = \tfrac{1}{2} \qquad \text{and} \qquad y = z = w = v = 0$$

and the asymptotic state is entangled with the negativity

$$N(\rho_{\mathrm{as}}) = \frac{\sqrt{2}-1}{4}. \tag{3.3}$$

Similarly, the state $|2_{\mathrm{A}}\rangle \otimes |3_{\mathrm{B}}\rangle$ has the population $\frac{1}{2}$ in the state $|a_{23}\rangle$ and also produces an asymptotic state with the same value of entanglement. The same behaviour can be observed for the initial states $|3_{\mathrm{A}}\rangle \otimes |1_{\mathrm{B}}\rangle$ and $|3_{\mathrm{A}}\rangle \otimes |2_{\mathrm{B}}\rangle$.

When the two atoms are initially in different excited states, i.e. we have the states $|1_{\mathrm{A}}\rangle \otimes |2_{\mathrm{B}}\rangle$ or $|2_{\mathrm{A}}\rangle \otimes |1_{\mathrm{B}}\rangle$, then the initial populations in the states $|a_{13}\rangle$ and $|a_{23}\rangle$ are equal to zero, but the population in the non-stable state $|a_{12}\rangle$ is nonzero and equals $\frac{1}{2}$. During the evolution this population is transformed into the states $|a_{13}\rangle$ and $|a_{23}\rangle$ in such a way that the asymptotic state ρ_{as} satisfies

$$\langle a_{12}|\rho_{\mathrm{as}}|a_{12}\rangle = 0$$

and

$$\langle a_{13}|\rho_{\mathrm{as}}|a_{13}\rangle = \langle a_{23}|\rho_{\mathrm{as}}|a_{23}\rangle = \tfrac{1}{4}.$$

Thus the state ρ_{as} is also entangled, but its negativity is less than (3.3) and equals $(\sqrt{6}-2)/8$.

Interesting examples of a pure non-diagonal initial state are given by the following superpositions of states $|1_{\mathrm{A}}\rangle \otimes |2_{\mathrm{B}}\rangle$ and $|1_{\mathrm{A}}\rangle \otimes |3_{\mathrm{B}}\rangle$:

$$\Psi_\phi = \cos\phi|1_{\mathrm{A}}\rangle \otimes |2_{\mathrm{B}}\rangle + \sin\phi|1_{\mathrm{A}}\rangle \otimes |3_{\mathrm{B}}\rangle, \qquad \phi \in [0, \pi/2]. \tag{3.4}$$

In that case, the initial populations

$$\langle a_{13}|P_{\Psi_\phi}|a_{13}\rangle = \tfrac{1}{2}\sin^2\phi, \qquad \langle a_{23}|P_{\Psi_\phi}|a_{23}\rangle = 0$$

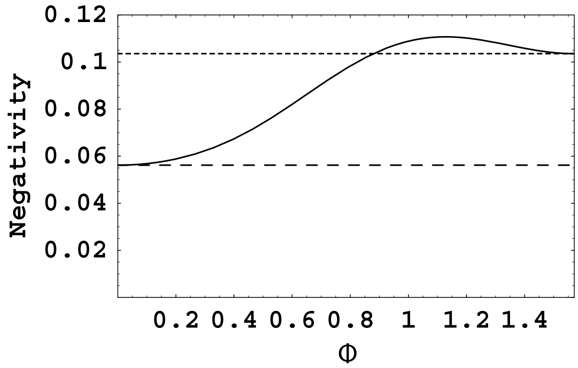

Figure 1. Asymptotic negativity of (3.4) as a function of ϕ. The values $(\sqrt{6}-2)/8$ and $(\sqrt{2}-1)/4$ are indicated by dashed and dotted lines, respectively.

and

$$\langle a_{12}|P_{\Psi_\phi}|a_{12}\rangle = \frac{1}{2}\cos^2\phi$$

are transformed into

$$\langle a_{13}|\rho_{\rm as}|a_{13}\rangle = \frac{1}{2}\sin^2\phi + \frac{1}{4}\cos^2\phi = \frac{1}{8}(3-\cos 2\phi),$$
$$\langle a_{23}|\rho_{\rm as}|a_{23}\rangle = \frac{1}{4}\cos^2\phi,$$
$$\langle a_{12}|\rho_{\rm as}\langle a_{12}| = 0.$$

In that case, the asymptotic states are parametrized by

$$x = \frac{1}{16}(3-\cos 2\phi), \qquad y = \frac{1}{8}\cos^2\phi, \qquad t = \frac{1}{2}, \qquad v = -\frac{1}{8}\sin 2\phi$$

and their negativity can be computed numerically. In figure 1, we plot the asymptotic negativity as a function of ϕ. This figure shows that for some values of the parameter ϕ superposition (3.4) can have larger asymptotic negativity than that achieved by the initial state $|1_{\rm A}\rangle\otimes|3_{\rm B}\rangle$, which has the maximal value of negativity produced by the dynamics for pure diagonal initial states.

3.2. Some mixed separable initial states

For the incoherent mixtures of pure states $|1_{\rm A}\rangle\otimes|2_{\rm B}\rangle$ and $|1_{\rm A}\rangle\otimes|3_{\rm B}\rangle$, i.e. the initial states

$$\rho = p|1_{\rm A}\rangle\otimes|2_{\rm B}\rangle\langle 1_{\rm A}|\otimes\langle 2_{\rm B}| + (1-p)|1_{\rm A}\rangle\otimes|3_{\rm B}\rangle\langle 1_{\rm A}|\otimes\langle 3_{\rm B}|, \tag{3.5}$$

the dynamics also produces entangled asymptotic states. One can check that their negativity is given by

$$N(\rho_{\rm as}) = \frac{1}{\sqrt{32}}\sqrt{4-2p+p^2} - \frac{1}{4} \tag{3.6}$$

and observe that (3.6) as the function of mixing parameter p is convex, so in contrast to the coherent superposition, the asymptotic negativity never exceeds (3.3). Analogous properties of negativity can be found for another mixtures of two pure diagonal states (figure 2).

To study the asymptotic entanglement produced for general class of mixed diagonal states, we consider two parameters to describe asymptotic states: their negativity and the degree of mixture given by the linear entropy

$$S_L(\rho) = \frac{9}{8}\,{\rm tr}(\rho-\rho^2). \tag{3.7}$$

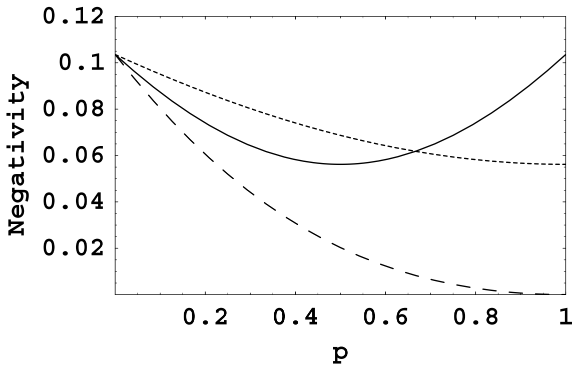

Figure 2. Asymptotic negativity for the mixtures of states $|1_A\rangle \otimes |2_B\rangle$ and $|1_A\rangle \otimes |3_B\rangle$ (dotted line), $|1_A\rangle \otimes |1_B\rangle$ and $|1_A\rangle \otimes |3_B\rangle$ (dashed line) and $|1_A\rangle \otimes |3_B\rangle$ and $|2_A\rangle \otimes |3_B\rangle$ (solid line).

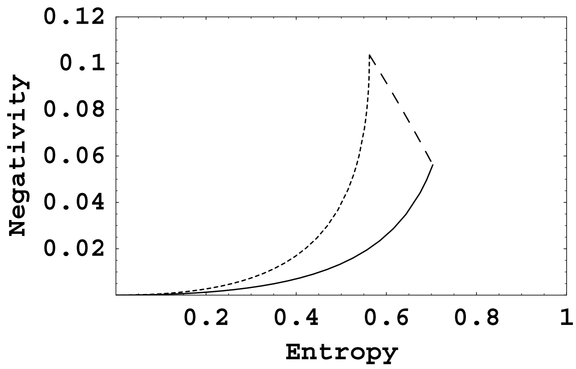

Figure 3. The set Λ_{as} for the asymptotic states for all diagonal initial states. The solid line corresponds to curve (3.8), the dotted line corresponds to (3.9) and the dashed line corresponds to (3.10).

As we show numerically, the set of all asymptotic states corresponding to the diagonal initial states is represented on the entropy-negativity plane by the region Λ_{as} bounded by three curves (figure 3). The boundary curves can be found analytically and are given by the following equations: the solid line in figure 3 is described by

$$N = \frac{1}{8}\sqrt{2s} - \frac{1}{4}\left(1 + \frac{\sqrt{3s-8}-1}{3}\right), \qquad S_L = \frac{9}{64}(8-s) \tag{3.8}$$

for $s \in [3, 8]$, the dotted line is given by

$$N = \tfrac{1}{4}(\sqrt{2s} - \sqrt{s-1} - 1), \qquad S_L = \tfrac{9}{16}(2-s) \tag{3.9}$$

for $s \in [1, 2]$, and finally, the dashed line is described by equations

$$N = \tfrac{1}{8}\sqrt{2s} - \tfrac{1}{4}, \qquad S_L = \tfrac{9}{64}(8-s), \tag{3.10}$$

where $s \in [9/16, 45/64]$.

Every asymptotic state corresponding to the diagonal initial state is given by some point from the set Λ_{as}. In particular, on curve (3.8) lie asymptotic states produced from the mixture of the states $|1_A\rangle \otimes |1_B\rangle$ and $|1_A\rangle \otimes |2_B\rangle$. Asymptotic states corresponding to the mixture of $|1_A\rangle \otimes |1_B\rangle$ and $|1_A\rangle \otimes |3_B\rangle$ lie on curve (3.9), whereas on curve (3.10) lie asymptotic states obtained from the mixture of states $|1_A\rangle \otimes |2_B\rangle$ and $|1_A\rangle \otimes |3_B\rangle$.

An example of the state lying inside the set Λ_{as} is given by the asymptotic state generated from the initial state

$$\rho_\infty = \tfrac{1}{9}\mathbb{1}_9 \tag{3.11}$$

which is maximally mixed state of two qutrits. One can check that the asymptotic state is given by formula (2.8) with $x = y = 1/12$ and $t = 2/3$. The corresponding values of linear entropy and negativity are equal to $9/16$ and $(3\sqrt{2} - 4)/12$, respectively. As we see, even in that case, the incoherent process of spontaneous emission produces such correlations which entangle two atoms and diminish the entropy of the system.

3.3. Initial states with bound entanglement

Consider now the following initial states:

$$\rho_a = \frac{1}{8a+1}\begin{pmatrix} a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 \\ a & 0 & 0 & 0 & a & 0 & \beta & 0 & \alpha \end{pmatrix}, \tag{3.12}$$

where

$$\alpha = \frac{1+a}{2}, \qquad \beta = \frac{\sqrt{1-a^2}}{2}$$

and $0 < a < 1$. The states (3.12) have positive partial transposition, but nevertheless are entangled [10]. Their entanglement is bound and cannot be distilled [11]. It can be checked by applying, for example, the realignment criterion of entanglement [17, 18]. The criterion can be stated as follows: if the trace norm of the realigned state $R(\rho)$ is greater than 1, the state ρ is entangled. We can also introduce the measure of entanglement based on this criterion. The so-called realignment negativity [19] $N_R(\rho)$ of the state ρ is defined by the formula

$$N_R(\rho) = ||R(\rho)|| - 1.$$

This measure can detect some bound entangled states, but not all of them. In the case of states (3.12), the values of $N_R(\rho_a)$ are contained in the interval $(0, 0.0035)$ and the maximal value is attained for $a = \frac{1}{4}$.

Numerical analysis of the evolution of the initial states (3.12) indicates that their realignment negativity very rapidly goes to zero, but for the later times the states become entangled with positive values of negativity. Thus the dynamics studied in the paper has a remarkable property: bound entangled initial states (3.12) evolve into 'free' entangled asymptotic states. By a direct calculation one can show that the asymptotic states have the form (2.8) with parameters

$$x = \frac{5a+1}{64a+8}, \qquad y = \frac{3a}{32a+4}, \qquad w = -\frac{\sqrt{1-a^2}}{32a+4}, \qquad t = \frac{21a+3}{32a+4}.$$

The values of negativity of those asymptotic states are plotted in figure 4.

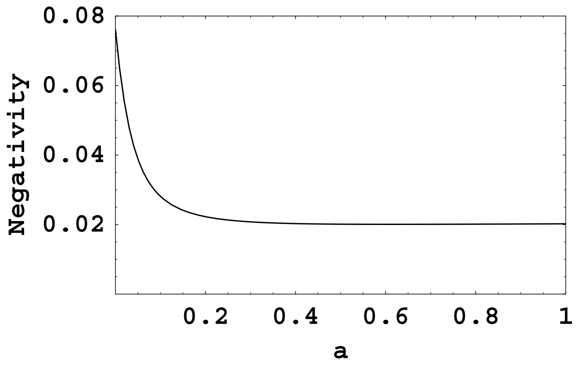

Figure 4. Asymptotic negativity of the initial states (3.12) as a function of the parameter a.

4. Conclusions

We have studied entanglement production in the system of two three-level atoms in the V configuration, coupled to the common vacuum. In the case of small (compared to the radiation wavelength) separation between the atoms, the system has nontrivial asymptotic states which can be entangled even if the initial states were separable. Particular examples of such separable initial states are pure states in which the atoms are in different excited states. The process of the photon exchange between the atoms produces correlations that entangle two atoms. It is interesting that when we superpose such initial states, we can enlarge the amount of the asymptotic entanglement. We have also characterized the entanglement of asymptotic states for mixed diagonal initial states. Using the description of mixed states in terms of the degree of mixture and entanglement, we have found the region on the mixture-entanglement plane corresponding to such asymptotic states. We have also shown by considering a specific example that the dynamical evolution of that system brings bound entangled PPT states into 'free' entangled NPPT asymptotic states.

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